

general
ans

2 Tangent + Arc
P1P2

we are using
The quadrilateral
To find

$\angle P_1EP_2$ angle
to get

$\angle P_1OP_2$ angle
& solve

$$\overline{OA} = d_1 = \sqrt{(x_1 - a)^2 + (y_1 - b)^2}$$

$$\overline{OB} = d_2 = \sqrt{(x_2 - a)^2 + (y_2 - b)^2}$$

$$\cos \theta_1 = \frac{r}{d_1}$$

$$\cos \theta_2 = \frac{r}{d_2}$$

$$\text{Tangent length} \Rightarrow (x_1 - a)^2 + (y_1 - b)^2 - r^2 = \overline{AP_1}^2$$

$$\Rightarrow \sqrt{(x_1 - a)^2 + (y_1 - b)^2 - r^2} = \overline{AP_1}$$

$$\overline{BP_2} = \sqrt{(x_2 - a)^2 + (y_2 - b)^2 - r^2}$$

for finding $\angle P_1OP_2$ (using AO, BO & finding $\angle AOB$ The using quadrilateral identity and then moving)

$$\text{form} \rightarrow \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\Rightarrow m_1 = \frac{y_1 - b}{x_1 - a} \quad \text{①}$$

$$m_2 = \frac{y_2 - b}{x_2 - a} \quad \text{②}$$

giving

$\tan \alpha$ (Assumed name of $\angle AOB$)

$$= \left| \frac{(x_1 - a)(y_1 - b) - (y_1 - b)(x_2 - a)}{(x_1 - a)(x_2 - a) + (y_1 - b)(y_2 - b)} \right|$$

or can we $\Rightarrow$ Dot product form

$$\text{giving us } \Rightarrow \cos \theta = \frac{(x_1 - a)(x_2 - a) + (y_1 - b)(y_2 - b)}{\sqrt{(x_1 - a)^2 + (y_1 - b)^2} \cdot \sqrt{(x_2 - a)^2 + (y_2 - b)^2}}$$

$$\therefore \text{Arc length} = r\theta$$

$$\text{and } \theta \Rightarrow (\alpha - \theta_3 - \theta_4)$$

$$\therefore \overline{AP_1} + \text{Arc} + \overline{BP_2}$$

$$\Rightarrow L_{\min} = \sqrt{d_1^2 - r^2} + \sqrt{d_2^2 - r^2}$$

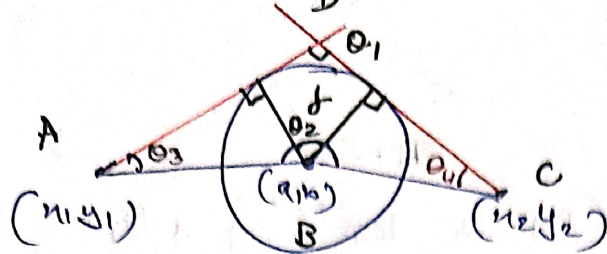
$$+ r \left[\alpha - \cos^{-1} \left(\frac{d}{r_1} \right) + \cos^{-1} \left(\frac{d}{r_2} \right) \right]$$

where

$$d_1^2 = (x_1 - a)^2 + (y_1 - b)^2$$

$$d_2^2 = (x_2 - a)^2 + (y_2 - b)^2$$

$$\alpha = \cos^{-1} \left(\frac{(x_1 - a)(x_2 - a) + (y_1 - b)(y_2 - b)}{d_1 d_2} \right)$$



first Attempt
Not final

Let's take that $r = \text{radius}$

$$\therefore \cos \theta_3 = \frac{r}{\text{hypotenuse}} \quad \cos \theta_4 = \frac{r}{\text{hypotenuse}}$$

$$\Rightarrow \cos \theta_3 = \frac{r}{AB} \quad \cos \theta_4 = \frac{r}{BC}$$

$$\therefore \cos \theta_3 = \frac{r}{\sqrt{(x_1 - a)^2 + (y_1 - b)^2}}$$

$$\cos \theta_4 = \frac{r}{\sqrt{(x_2 - a)^2 + (y_2 - b)^2}}$$

Using ~~vector~~ eqn $\Rightarrow$ ~~Calculate~~ slope of AB $\Rightarrow m_{AB} = \frac{y_1 - b}{x_1 - a}$

$$m_{BC} = \frac{y_2 - b}{x_2 - a} \neq \left[\frac{y_2 - b}{x_2 - a} = m_{BC} \right] \left[m_{AB} = \frac{y_1 - b}{x_1 - a} \right]$$

$$\therefore \tan \theta_2 = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right| = \left| \frac{\frac{y_2 - b}{x_2 - a} - \frac{y_1 - b}{x_1 - a}}{1 + \frac{(y_2 - b)(y_1 - b)}{(x_2 - a)(x_1 - a)}} \right|$$

$$\Rightarrow \tan \theta_2 = \left| \frac{(x_1 - a)(y_2 - b) - (y_1 - b)(x_2 - a)}{(x_2 - a)(x_1 - a) + (y_2 - b)(y_1 - b)} \right|$$

Since we have 3 angles
we can get $\Rightarrow$

$$\cos^{-1}(A) + \cos^{-1}(B) + \tan^{-1}(C) + \theta_1 = \cancel{2\pi} \pi$$

$$\therefore \theta_1 = \pi - (A) - (B) - (C)$$

$$\therefore \theta_1 + \pi + (\theta_2) = 2\pi$$

$$\pi - (A) - (B) - (C) + \theta_2 = 2\pi$$

①

using this $\Rightarrow$ $\theta_2 = \cos^{-1}(\quad) + \cos^{-1}(B) + \tan^{-1}(C)$

$\therefore$ radius $\Rightarrow$

$$\sqrt{(x^2 - a)^2 + (y - b)^2} + r^2 = D_1$$

$$\sqrt{(x_2 - a)^2 + (y_2 - b)^2} - r^2 = D_2$$